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# ESTIMATION OF EVAPORATION FROM HORIZONTAL SURFACES

**M. M. SHAH, P.E.**
*Member ASHRAE*

ABSTRACT

Many practical applications require the estimation of rate of evaporation from horizontal surfaces. Such problems are encountered in many fields of engineering including HVAC, hydrology and chemical engineering. HVAC calculations are done almost exclusively by the empirical Carrier equation which is unverified. In hydrology a number of empirical equations are used. General equations were developed based on the analogy between heat and mass transfer. The various empirical equations for evaporation from water surfaces were compared with each other and the analogy equations for a wide range of parameters. The Carrier equation predictions at higher velocities are far higher than all other equations while the latter substantially agree with each other.

The dimensionless equations are also applicable to systems other than air-water and hence would be useful in a variety of problems. Simplified dimensional versions of these equations are also presented which apply to only air-water systems. A formal derivation of the Lewis relation for free convection evaporation is also given.

## INTRODUCTION

Calculating the rate of evaporation from flat horizontal surfaces is often required of HVAC engineers as well as engineers concerned with other disciplines ( e.g. hydrology and chemical engineering ). Among problems handled by HVAC engineers are: evaporation of water from swimming pools, spent nuclear fuel pools, and process tanks. Hydrologic engineers are interested in determining the rate of evaporation from water reservoirs. Chemical engineers need to know the rate of evaporation of a variety of liquids and solutions into air and other gases. This paper particularly adresses HVAC engineering problems. However, much of the material is also applicable to other engineering disciplines.

A study of books normally referred to by HVAC engineers revealed that the only guidance available for calculating evaporation rates is given in the ASHRAE Handbook[1]. It recommends the use of the Carrier equation[2] for estimating evaporation from swimming pools. Note that Bishop and Daley[10] presented a nomogram based on the Carrier equation and for this reason the Carrier equation is sometimes known as the Bishop and Daley equation. Due to the absence of any other guidence, virtually all HVAC engineers use the Carrier equation for estimating evaporation from swimming pools as well as other horizontal water surfaces. However the reliability of this equation is open to question since Carrier[2] gave no details of his experiments in his paper, nor could any other report of theoretical or experimental verification of this equation be

M. Mohammed Shah, Consulting Engineer, 15, Rush Street, Port Jefferson Station, New York 11776.

found.

Widespread use of an unconfirmed predictive technique is clearly undesirable. Hence the author undertook a study whose primary objective was to critically evaluate the Carrier equation. The Carrier equation was compared with some empirical equations used in hydrology for calculating evaporation from water reservoirs. Furthermore, generalized equations for calculating evaporation were developed by using the analogy between heat and mass transfer. Comparison of these various predictive techniques suggests that the Carrier equation is most probably unreliable.

The dimensionless mass transfer equations presented here are applicable not only to evaporation of water into air but of any other substance ( including sublimating solids ) into any gas. Furthermore, simplified dimensional equations applicable to air-water systems are presented which the practical engineers ( without knowledge of heat and mass transfer ) can use. These equations have been derived from the dimensionless mass transfer equations through reasonable assumptions and are comparable in accuracy. A derivation is also presented to show that the Lewis relation is applicable to free convection mass transfer. Finally, the comparison between the mass transfer equations ( based on the analogy with heat transfer ) and the empirical correlations used in evaporation calculations for reservoirs would be of interest to civil and hydrologic engineers.

For brevity, the analogy between heat and mass transfer has been referred to simply as "analogy" throughout this paper. Similary, the equations based on the analogy between heat and mass transfer have been referred to as "analogy equations."

CARRIER'S EQUATION AND EXPERIMENTS

Carrier presented the following empirical equation for flow of air parallel to horizontal water surface:

$$G = \frac{(0.319 + 0.0047V)}{\lambda} \quad (p_w - p_{we}) \tag{1}$$

Carrier noted that "the way the air current is applied to the surface will make a very marked difference in the coefficient of velocity." It was stated that a large number of tests were made, a wide range of water temperatures was used, and that the temperature and humidity of air were automatically regulated. From Fig. 4 of Ref 2, it appears that the air velocity varied from 0 to 427 m/min. No other experimental details are given. The size of the water pool, method of air distribution, the location where air velocity was measured, instrument accuracy, and deviation of data from correlation are all unknown. In applying Eq 1, it is generally assumed that V is the free stream velocity and constant along the length of water surface. In the absence of any information to the contrary, the present author has also made the same assumption.

PREDICTIVE EQUATIONS USED IN HYDROLOGY

For designing and analyzing water reservoirs, the loss of water due to evaporation has to be estimated. Many equations for this purpose have been proposed. Most of these follow Dalton's law of diffusion[3] which may be stated as:

$$G = F\ (p_w - p_{we}) \tag{2}$$

where F is a coefficient whose value depends on factors such as wind velocity. It will be noted that the Carrier equation, Eq 1, is also of the same form.

Hjelmfelt and Cassidy[4] listed a number of empirical equations used in hydrology, and two of these have been chosen for comparison. The first one is the equation of Meyer[5] which may be written as:

$$G = 15.5 \times 10^{-5} (1+.0036V_{30})\ (p_w - p_{we}) \tag{3}$$

check

$V_{30}$ is the air velocity 30 ft (9.14m ) above the water surface. The coefficient 0.000155 in Eq 3 is for small shallow lakes. For large deep lakes, the coefficient becomes 0.000115. As the quantitative relation between size and the coefficient is not known, a coefficient of 0.000155 has been used throughout for consistency.

A more recent correlation is the Lake Hefner equation proposed by Harbeck and Anderson[6] which may be written as:

$$G = 10^{-7} (212 + 6.85V_{13}) (p_w - p_{we}) \tag{4}$$

$V_{13}$ is the velocity 13 ft (3.96m ) above the water surface.

Another method used widely in hydrology is what may be called the pan coefficient method. The rate of evaporation from a satndard pan containing water is measured while it is exposed to wind near the reservoir site. The rate of evaporation from the reservoir is then estimated as follows:

$$G_{reservoir}/G_{pan} = \text{pan coefficient} \tag{5}$$

The subcripts are self-explanatory. According to Hjemfelt and Cassidey[4], the pan coefficient generally varies from 0.6 to 0.8. The standard pan is 1.22 m in diameter and o.25m deep.

EQUATIONS BASED ON ANALOGY

It is well established that the transfer of heat and mass are analogous. As pointed out by Eckert and Drake[9], among many others, an equation for free convection heat transfer may be applied for free convection mass transfer simply by replacing the Nusselt number by the Sherwood number, the Raleigh number by the mass transfer Raleigh number, and the Prandtl number by the Schmidt number. Furthermore, an equation for forced convection heat transfer may be used for forced convection mass transfer by replacing the Nusselt number by the Sherwood number and the Prandtl number by the Schmidt number.

The water surface may be considered to be a horizontal plate heated on its upper surface. Equations for heat transfer during free and forced convection over horizontal plates are given in many books, such as the one by Holman[8]. These heat transfer equations were converted to mass transfer equations by using the method outlined in the previous paragraph. The dimensionless mass transfer equations thus derived are given here. Their derivation is explained in Appendix 1.

The mass transfer coefficient can be defined in many ways. Here we define it by the following equation:

$$G = h_M \rho (w_i - w_e) \tag{6}$$

The various equations based on analogy for determining $h_M$ are given in the following:

For laminar free convection ( $Gr_M\ Sc < 2 \times 10^7$ )

$$\frac{h_M L}{D} = 0.54 \left[\frac{(\rho_e - \rho_i) g L^3}{\mu D}\right]^{1/4} \tag{7}$$

For turbulent free convection ($Gr_M\ Sc > 2 \times 10^7$)

$$\frac{h_M L}{D} = 0.14 \left[\frac{(\rho_e - \rho_i) g L^3}{\mu D}\right]^{1/3} \tag{8}$$

For forced laminar flow ($Re_L < 5 \times 10^5$) parallel to the surface of plate, the mean Sherwood number for the plate of length L is given by:

$$Sh_M = 0.664\ Sc^{1/3}\ Re_L^{1/2} \quad (9)$$

For forced flow when $Re_L > 5 \times 10^5$, the boundry layer is laminar over the first part of plate where the Reynolds number $< 5 \times 10^5$ and is turbulent over the rest of the plate. For such a case, the mean Sherwood number is given by:

$$Sh_m = Sc^{1/3}\ (0.036\ Re_L^{0.8} - 836) \quad (10)$$

The forced convection expressions, Eq 9 and 10, have been derived on the assumption that the free convection effects are neglegible. This assumption is correct only if the buoyancy forces are much smaller than the inertia forces. In those cases where buoyancy effects are not neglegible, free convection effects would enhance the mass transfer due to forced convection effects. Noting the fact that the Grashof number is the ratio of buoyancy forces to viscous forces and the Reynolds number is the ratio of interia forces to viscous forces, the boundaries between the regimes of free, forced, and mixed convection are likely to be determined by the relative magnitude of Grashof and Reynolds numbers. Many theoretical and experimental studies have been carried out, ( e.g. Ref 12 and 13 ). However, the problem has as yet not been fully resolved. As a first approximation, the author assumed that the forced and free convection effects are independent of each other and additive. Thus the total evaporation from a surface can be calculated by calculating the evaporation due to free and forced convection separately and adding them together. The author believes that the actual evaporation would not exceed that calculated with this assumption.

For gas-liquid systems having the Lewis number approx. equal to 1, the analogy between heat and mass transfer leads to the following equation:

$$h_M = h/\rho\ C_p \quad (11)$$

Eq 11 is known as the Lewis relation. Its derivation for forced convection flow is given in most text books, including Ref 8 and 9. However, it is also valid for free convection. As the derivation for free convection could not be found in any book, it is given here in Appendix 1.

As the Lewis number for air-water is close to 1, Eq 11 could be used without significant error. However, all calculations for Fig. 1 to 11 have been done with Eq 7 to 10.

Kusuda[14] has stated that the properties of air should be calculated at the interfacial conditions. Calculations using the mean of properties at the interface and in free stream gave virtually the same results as those with properties evaluated at interfacial conditions.

## COMPARISON BETWEEN VARIOUS EQUATIONS

Evaporation rates were calculated by the Carrier equation, Meyer equation, Lake Hefner equation, and the analogy equations over a wide range of air velocities, water pool length, and water temperatures. The complete range of parameters analyzed is as follows:

| | |
|---|---|
| Water temperature | 15.5 to 65.5° C |
| Free stream velocity of air | 0 to 610 m/min |
| Length of water pool | 3 to 305 m |
| Free stream air condition | 15.5° C dry and saturated |

In doing these calculations with the analogy equations, it was assumed that the air at the interface with water is saturated at the temperature of water. This assumption is quite reasonable as the air at interface would be heated by direct contact with water as well as gain heat and moisture from the vapor leaving the water surface.

In calculations with the lake Hefner equation, it was assumed that $V_{13}$ is the free stream velocity. Similarly, in using the Meyer equation, $V_{30}$ was assumed to be the free stream velocity.

The results of these calculations are shown in Fig. 1 to to 11 and are discussed in the following. In order to give an idea of the relative magnitude of the rate of evaporation due to free and forced convection calculated by analogy equations, both the total evaporation rate as well as the evaporation rate due to forced convection alone have been shown in the figures.

Evaporation By Free Convection Alone

Fig. 1 and 2 show the comparison of the various equations at zero air velocity. The evaporation is purely due to free convection. It was found that ( Gr Sc ) was always well into the turbulent range even for a length of 3m. Hence Eq 8 was used throughout to calculate the evaporation according to the analogy between heat and mass transfer. In Eq 8, L cancels out from both sides and hence its predictions are independent of length. The Carrier equation and the Lake Hefner correlation do not contain L as a parameter. The coefficient in Meyer equation decreases from $15.5 \times 10^{-5}$ to $11.5 \times 10^{-5}$ as the size of reservoir increases.

Fig. 1 is based on calculations with air assumed dry at 15.5 °C while Fig. 2 is for air saturated at 15.5 °C. The results at both air conditions are qualitatively the same. This suggests that relative humidity does not affect the accuracy of the various methods of prediction. The Meyer and Carrier equations are in close agreement throughout the range. The analogy predictions are also in agreement for water temperatures above 30 °C. For 15 °C water and dry air, the analogy prediction is less than half the predictions of the Meyer and Carrier equations. The predictions of the Lake Hefner equation are far lower than any of the other equations at any water temperature.

As the wind velocity in an open area is rarely zero for any length of time, it would be unrealistic to expect any empirical equation based on the data for wind exposed resorvoirs to make good predictions at zero velocity. Hence the complete disagreement of the Lake Hefner correlation is not surprising and the agreement between the equations of Carrier and Meyer may be merely fortuitous. However, the agreement between the analogy and the Carrier equations at larger temperature differences is reassuring. At lower temperature differences, the author believes that the analogy predictions are probably more accurate. However, this belief needs confirmation through experimental measurements.

A comparison of Fig. 1 and 2 leads to the conclusion that the relative humidity of free stream air does not affect the comparison between different calculation methods. Hence this parameter was not explored any further. The remainder of the calculations were done for dry air.

Effect Of Free Stream Velocity

The effect of air velocity is shown in Fig. 3, 4, and 5. In all these figures, calculations have been done assuming L = 30.5m and free stream air dry at 15.5 °C. Fig. 3, 4, 5 are respectively for water temperatures of 15.5, 32.2 and 65.5 °C. It is seen that above a velocity of about 150 m/min, the predictions of Carrier equation are far higher than those of any of the other equations. On the other hand, the predictions of the analogy agree well with the Meyer equation above a velocity of 150 m/min. As this would be about the minimum average wind velocity over an extented period of time, the Meyer equation agrees with the analogy in the velocity range in which the former can reasonably be expected to be accurate. The Lake Hefner correlation is in good agreement with the Meyer equation and the analogy equations above a velocity of about 300 m/min. This suggests that the data on which the Lake Hefner correlation was based was probably mostly confined to higher velocities. It is also interesting to note that the predictions of the Lake Hefner correlation are quite close to the analogy predictions for forced convection alone for all velocities.

## Effect Of Length

The effect of length on free convection evaporation rate has already been discussed. To investigate the effect of length on forced convection evaporation, calculations were done for lengths between 3 and 300 m at various water temperatures and air velocities. Some representative results are shown in Fig. 6, 7, and 8. These figures show that the evaporation rate according to the analogy decreases with length: the higher the velocity, the greater the decrease in predicted evaporation rate. The Meyer equation also predicts a decrease in evaporation rate with increasing size of the reservoir. The Carrier and Lake Hefner correlations do not include length as a parameter. It will be noted that the analogy predicts only a slight decrease in evaporation as the length increases from 3 to 30 m. As the Carrier equation is basically intended for HVAC engineers whose work is mostly concerned with smaller sizes, the absence of the length as a parameter is in agreement with the analogy predictions. The agreement between the Lake Hefner correlation and the analogy predictions is seen to be better at larger lengths. This is reasonable as the Lake Hefner correlation is intended for large reservoirs.

Fig. 7 and 8 for velocities representative of actual wind velocities. In these figures, it is noted that the ratio of the rate of evaporation for 300 m length to that for 3 m length is between 0.6 and 0.8. These figures also indicate that the evaporation rate for 1.22 m length would be virtually the same as for 3 m length. Furthermore a 300 m length is representative of actual reservoirs. Hence the analogy equations predict pan coefficients ( Eq 5 ) in the range found from actual field data[4]. This agreement is very reassuring.

## Effect Of Water Temperature

Effect of water temperature during free convection evaporation has been discussed earlier in the paper, and it was noted that the Carrier equation appears to overpredict for small temperature differences between air and water. The effect of water temperature during forced convection can be studied in Fig. 3 to 8. Fig. 9, 10 and 11 show this effect more clearly. At a velocity of 30 m/min, the results as shown in Fig. 9 are quite similar to those at zero velocity. Fig. 11 shows that , at a velocity of 610 m/min, the Meyer equation, Lake Hefner equation and analogy predictions are in agreement at all temperatures. However, the Carrier equation predicts 2 to 3 times higher than other equations at all water temperatures and is therefore suspected to be in error.

## SUMMARY OF RESULTS

1. At air velocities higher than about 150 m/min, the Carrier equation predicts considerably higher than any of the other equations at all water temperatures.

2. The Meyer equation and analogy predictions are in good agreement above a velocity of 150 m/min for all values of other parameters.

3. The Lake Hefner equation agrees fairly well with the Meyer equation and analogy predictions above 300 m/min velocity.

4. At zero velocity, the predictions of the Carrier equation, the Meyer equation, and the analogy agree closely for temperature difference between air and water greater than 15 °C. For lower temperature differences, the Carrier and Meyer equations agree but the analogy predictions are lower. The same results are obtained with low-velocity forced convection.

5. For free convection and low-velocity forced convection, the predictions of of the Lake Hefner correlation are far lower than those of other methods at all water temperatures.

6. The pan coefficients predicted by the analogy are in good agreement with field measurements.

## CONCLUSIONS

Final conclusions regarding any predictive method can be reached only through direct comparison with a wide variety of experimental and field data. The following tentative conclusions are reached on the basis of comparisons presented in the following:

1. The Carrier equation is not reliable for air velocities above 150 m/min.

2. The Carrier equation is reliable for zero and low velocities when the difference in temperature of air and water is 15 °C or greater but is of questionable accuracy for smaller temperatures differences.

3. The Lake Hefner correlation is relaible for conditions for which it is intended. For air velocities less than 300 m/min or water resorvoirs less than 30 m long, it predicts too low.

4. The Meyer correlation is reliable above a velocity of about 150 m/min. At lower velocities, its reliability is open to question.

5. The equations based on the analogy between heat and mass transfer are reliable at velocities above 150 m/min at any air-water temperature difference. At lower velocities, these are reliable when air-water temperature is greater than about 15 °C. Below this temperature difference, the reliability of analogy equations cannot be proved or disproved on the basis of the analyses performed here. However, as the analogy between heat and mass transfer is well established, the author believes that the analogy equations are reliable even at low temperature differences coupled with low velocities.

## APPLICATION TO HVAC SYSTEM

If air flow over the water surface is actually parallel to it and is at a uniform velocity, the evaporation rate can be calculated easily using the analogy equations or one of the other equations. The author recommends the Carrier equation at low velocities and the analogy equations at higher velocities. The Carrier equation is recommended at low velocities because it predicts higher than the analogy equations and the evidence examined here is not sufficient to indicate which one is more reliable.

In many practical HVAC systems, air flow is neither parallel to the water surface nor at a uniform velocity. For example, in push-pull systems, air is discharged by horizontal ducts at one end of the water pool and sucked into ducts at the other end. The air discharged by a duct expands into a cone and the velocity decreases rapidly with distance from discharge point. Thus the velocity field is three dimensional while the analogy equations presented here are based on the assumption that the velocity varies only in one direction ( normal to the water surface ). Hence these equations become inapplicable. However, if the heat transfer coefficient for such a three dimensional flow could be calculated, the mass transfer coefficient could be calculated by using the Lewis relation, Eq 11.

The Lake Hefner equation, as well as the Carrier and Meyer equations are also intended for uniform velocity parallel to water surface and hence are inapplicable to the push-pull system or other systems in which air flow is not one-dimensional. It is possible that some equivalent mean air velocity could be defined for which these equations would make correct predictions. For developing the method of calculating such an equivalent velocity, varied data on evaporation under the flow conditions of interest have to be analyzed. Such analyses are beyond the scope of this study. Until more research yields better estimation techniques, the use of the Carrier equation with integrated mean air velocity is suggested. The Carrier equation is suggested because its predictions are always higher than other methods and hence its use provides greater safety margin.

## CONCLUDING REMARKS

The primary objective of this paper was to critically evaluate the Carrier equation which is widely used for HVAC calculations but for which no verification is available in literature. Comparison with other predictive techniques including analogy between heat and mass transfer indicates that this equation is most probably inaccurate. A firm conclusion can be reached only after direct comparison with experimental data. It is hoped that such research will be done. Furthermore, experimental investigation is needed for those cases in which air is not parallel to the surface and its velocity is not uniform.

The results of comparing the Meyer and Lake Hefner equations as well as the pan coefficient method should be of interest to engineers concerned with water reservoir design. The good agreement with the predictions based on the analogy between heat and mass transfer at wind velocities of practical interest is reassuring. It shows that the evaporation from water reservoirs can be reliably calculated by the analogy equations, thus largely eliminating the need for empirical equations.

The assumption made in this paper that the forced and free convection effects are independent and additive is an over-simplification. It would lead to over-prediction in many cases. The true relationship needs to be dertermined through further research. Some guidance is available from studies ( e.g. Ref 12 and 13 ) but much remains to be done.

## APPENDIX 1

Derivation of Eq 7 to 11 through the analogy between heat and mass transfer is explained here.

### Free Convection Equations:

The water surface may be considered to be a horizontal flat plate with its upper surface heated. For heat transfer due to turbulent free convection from such a plate ( $Gr\, Pr > 10^9$ ), Ref 7 gives the following equation:

$$Nu = 0.14\ (Gr\, Pr)^{1/3} \tag{12}$$

Holman[8] recommends Eq 12 for horizontal square plates with ( Gr Pr ) between $2 \times 10^7$ and $3 \times 10^{10}$. For laminar free convection heat transfer, Ref 7 gives the following equation:

$$Nu = 0.54\ (Gr\, Pr)^{0.25} \tag{13}$$

Eckert and Drake[9] state that the equation for free convection mass transfer can be obtained from the corresponding heat transfer equation simply by replacing the Nusselt number by the Sherwood number, ( Gr Pr ) by ( $Gr_M\, Sc$ ), and the Prandtl number by the Schimdt number. Using this method, the mass transfer equation for turbulent free convection is:

$$Sh = \frac{h_M L}{D} = 0.14\ (Gr_M\, Sc)^{1/3} \tag{14}$$

The mass transfer Grashof Number $Gr_M$ is defined as[9]:

$$Gr_M = \frac{\gamma g\, (w_i - w_e)\, L^3 \rho^2}{\mu^2} \tag{15}$$

where:

$$\gamma = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial w}\right) \tag{16}$$

Eq 16 can be approximately written as:

$$\gamma = \frac{\rho_e - \rho_i}{\rho\, (w_i - w_e)} \tag{17}$$

Thus Eq 15 becomes:

$$Gr_M = \frac{(\rho_e - \rho_i)\, g L^3 \rho}{\mu^2} \tag{18}$$

Substituting Eq 18 in 14:

$$\frac{h_M L}{D} = 0.14 \left[\frac{(\rho_e - \rho_i)\, g L^3}{\mu D}\right]^{1/3} \tag{8}$$

Eq 7, for laminar free convection, can similarly be derived from Eq 13.

Lewis Relation For Free Convection:

The heat transfer Grashof number is defined as:

$$Gr = \frac{g \beta (t_i - t_e) L^3 \rho^2}{\mu^2} \tag{19}$$

where:

$$\beta = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial t}\right) \tag{20}$$

This can be approximated as:

$$\beta = \frac{1}{\rho} \left[\frac{\rho_e - \rho_i}{t_i - t_e}\right] \tag{21}$$

Substituting Eq 21 in Eq 19, we have:

$$Gr = \frac{(\rho_e - \rho_i)\, g L^3 \rho}{\mu^2} \tag{22}$$

Comparison of Eq 18 and 22 shows that:

$$Gr = Gr_M \tag{23}$$

Comparison of Eq 12, and 14 and using Eq 23:

$$\frac{Sh}{Nu} = \left(\frac{Sc}{Pr}\right)^{1/3} = (1/Le)^{1/3} \tag{24}$$

where Le is the Lewis number.

If Lewis number is 1, Eq 24 yields,

$$h_M = h\, D/k \tag{25}$$

when Le = 1, Sc = Pr and hence,

$$\mu/(\rho D) = C_p\, \mu/k \tag{26}$$

Hence Eq 25 becomes,

$$h_M = h/\rho\, C_p \tag{11}$$

Eq 11 is the Lewis relation whose derivation for forced convection can be found in most books. Here it has been shown that the Lewis relation is also valid for turbulent free convection. In the same way, the Lewis relation can be proved for laminar free convection.

Forced Convection Equations:

Holman[8] gives the following equation for laminar forced convection parallel to a horizontal heated plate:

$$Nu_x = 0.332\ Pr^{1/3}\ Re_x^{1/2} \tag{27}$$

$Nu_x$ is the local Nusselt number at a distance x from the leading edge where $Re_x$ is the length Reynolds number at the same location.

As was mentioned earlier, an equation for forced convection heat transfer can be converted to the corresponding mass transfer equation by replacing Nu by Sh and Pr by Sc. Hence the equation for mass transfer during forced laminar convection is:

$$Sh_x = 0.332\ Sc^{1/3}\ Re_x^{1/2} \tag{28}$$

It can be shown by integrating Eq 28 that the mean Sherwood number for a plate of length L would be:

$$Sh_m = 0.664\ Sc^{1/3}\ Re_L^{1/2} \tag{9}$$

For a plate of length L over which both laminar and turbulent boundry layars are present Holman[8] gives the following equation for mean Nusselt number:

$$Nu_m = Pr^{1/3}\ (0.036\ Re_L^{0.8} - 836) \tag{29}$$

Eq 29 was derived by integrating the expressions for local Nusselt numbers for laminar and turbulent flows, assuming the critical Reynolds number to be $5 \times 10^5$. Furthermore, the turbulent flow heat transfer equation ( not given here ) was derived by invoking analogy between heat and momentum transfer.

Invoking the analogy between heat and mass transfer, Eq 29 yields the following expression for mean Sherwood number:

$$Sh_m = Sc^{1/3}\ (0.036\ Re_L^{0.8} - 836) \tag{10}$$

Eq 29, and therefore Eq 10, is strictly applicable to $Re_L \le 10^7$ because of the expression for the turbulent friction coefficient used in its derivation. However, use to a somewhat higher $Re_L$ does not cause any significant error.

## APPENDIX 2

### SPECIALIZED FORMULAE FOR AIR-WATER MIXTURES

Many practical engineers are unfamiliar with heat and mass transfer terminology and would find it difficult to directly use the dimensionless equations based on the analogy between heat and mass transfer presented in the paper. For their convenience, the analogy equations have been converted to simple dimensional formulae applicable to air-water mixtures only. These are given here:

The evaporation due to free convection effects, $G_{free}$ is given by:

$$G_{free} = \left[85\ (0.0016t + 0.22)^{2/3}\ (\rho_e - \rho_i)^{1/3}\right] \rho\ (w_i - w_e) \tag{30}$$

The evaporation due to free convection effects, $G_{free}$ is given by:

$$G_{forced} = \frac{0.0016t + 0.22}{L} \left[2.54 \; (L \, V\rho)^{0.8} - 256\right] \rho (w_i - w_e) \quad (31)$$

The total evaporation due to combined forced and free convection is:

$$G = G_{free} + G_{forced} \quad (32)$$

If $G_{forced}$ calculated by Eq 31 is negative, assume it to be zero. Furthermore, the above equation are in SI units as listed in the "Nomenclature."

Eq 30 has been derived from Eq 6 and assuming constant dynamic viscosity and Schmidt number, and substituting an empirical equation for D in terms of t. The data for D were taken from Ref 11 and covered a range from 0 to 100 °C. Free convection is assumed turbulent as all calculation showed this to be the case. Eq 31 is derived from Eq 6 and 10 in the same way. Use of Eq 10 means that $Re_L$ was assumed to be in the turbulent range. This is justified on the ground that if $Re_L$ is in the laminar range, $G_{forced}$ will be negligible compared to $G_{free}$. Hence Eq 30, 31 and 32 can be used directly for calculating the evaporation rate without the need for calculations to determine whether the flow is turbulent or laminar.

Eq 29 31 and 30 are in SI units. In British units they may be written as:

$$G_{free} = 702\,(0.00087t + 0.192)^{2/3} \; (\rho_e - \rho_i)^{1/3} \; \rho (w_i - w_e) \quad (33)$$

$$G_{forced} = \frac{0.0092t + 2.07}{L} \left[3.5 \; (LV\rho)^{0.8} - 256\right] \rho \, (w_i - w_e) \quad (34)$$

The units in Eq 34 and 33 are:

$\rho$ in $lb/ft^3$ ( density of saturated air at water surface temperature )

w in lb/lb

t in F ( temperature of water surface )

L in ft

V in ft/min

G in $lb/h \; ft^2$

A word of caution regarding the definitions of $\rho$ and w is desirable. Most psychrometric charts and tables give the density as mass of dry air per unit volume. But $\rho$ is the total mass of air-water mixture per unit volume. Similarly, w is the mass of water vapor divided by the total mass of air-water mixture. Psychrometric tables generally give the humidity ratio W which is defined as the mass of water vapor divided by the mass of dry air alone. The relation between w and W is as follows:

$$w = \frac{W}{1 + W} \quad (35)$$

If $\rho_{psych}$ be the value of density from psychrometric tables, $\rho$ is calculated as:

$$\rho = (\,1 + W\,)\,\rho_{psych} \quad (36)$$

Another point to remember is that t and $\rho$ used in Eq 30, 31, 33 and 34 are evaluated at the conditions at the interface between water and air. These equations could be further simplified by assuming D and $\rho$ to be constant. However, this will cause large errors when temperature varies significantly from the reference temperature. Eq 30 to 34 are comparable in accuracy to the full dimensionless equation from 10 to 80 °C and yet are simple enough for quick calculations with pocket calculators.

## NOMENCLATURE

Units of measurement are given only for those symbols used in dimensional equations. Any consistent system of units can be used in the dimensionless equations.

| | |
|---|---|
| Cp | Specific heat of air at constant pressure |
| D | Coefficient of molecular diffusion between air and water |
| g | Accelaration due to gravity |
| G | Evaporation rate, kg/h $m^2$ of water surface |
| Gr | Heat transfer Grashof number, defined by Eq 19 and 21 |
| $Gr_M$ | Mass transfer Grashof number, defined by Eq 15 and 18 |
| h | Heat transfer coefficient |
| $h_M$ | Mass transfer coefficient, defined by Eq 6 |
| L | Length of water surface in the direction of air flow, m |
| Le | Lewis number, ( Pr/Sc ) |
| k | Thermal conductivity of air |
| Nu | Nusselt number, h L/k |
| $p_w$ | Pressure of saturated water vapor at the temperature of water, $N/m^2$ |
| $p_{we}$ | Partial pressure of water vapor in free stream air, $N/m^2$ |
| $p_{wi}$ | Partial pressure of water vapor in air at interface with water, $N/m^2$ |
| Pr | Prandtl number of air, Cp $\mu/k$ |
| $Re_L$ | Reynolds number of air, ( $\rho L V/\mu$ ) |
| Sc | Schmidt number, ( $\mu/\rho D$ ) |
| Sh | Sherwood number, ( $h_M L/D$ ) |
| $Sh_m$ | Mean Sherwood number for a plate of length L |
| t | Temperature of air, C |
| V | Free stream velocity of air, m/min |
| w | Concentration of water vapor in air, mass of water/mass of mixture |
| W | Humidity ratio of air, mass of water/mass of dry air |
| $\rho$ | Density of air-water mixture, mass of mixture/volume of mixture |
| $\rho_{psych}$ | Density according to psychrometric tables, mass of dry air/volume of mixture. |
| $\lambda$ | Latent heat of vaporization of water, kJ/kg |
| $\mu$ | Dynamic viscosity of air |

Subscripts

| | |
|---|---|
| m | Mean value for a surface of length L |

e In the free stream

i At the interface

free Due to free convection effects

forced Due to forced convection effects

Abbreviation

FC Due to forced convection

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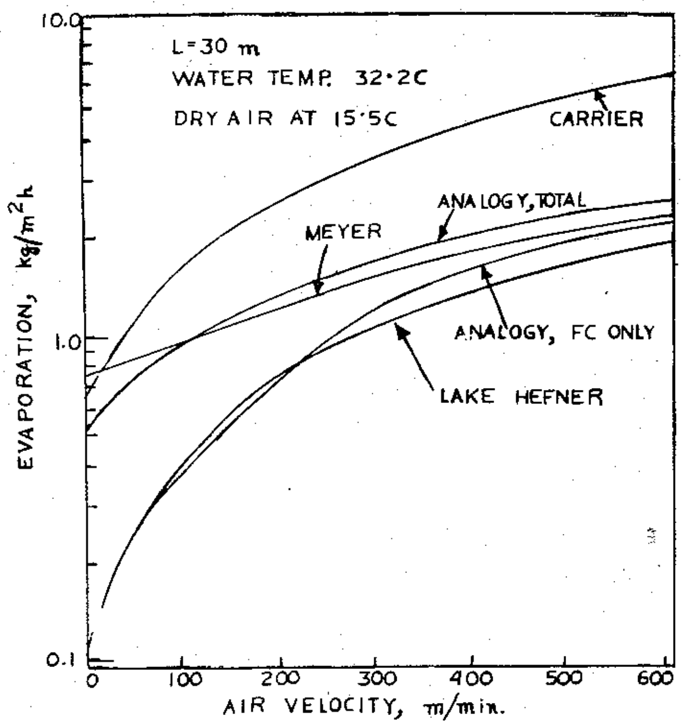


*Fig. 4 Effect of air velocity on the rate of evaporation calculated by various methods; water temperature is 15.5°C*

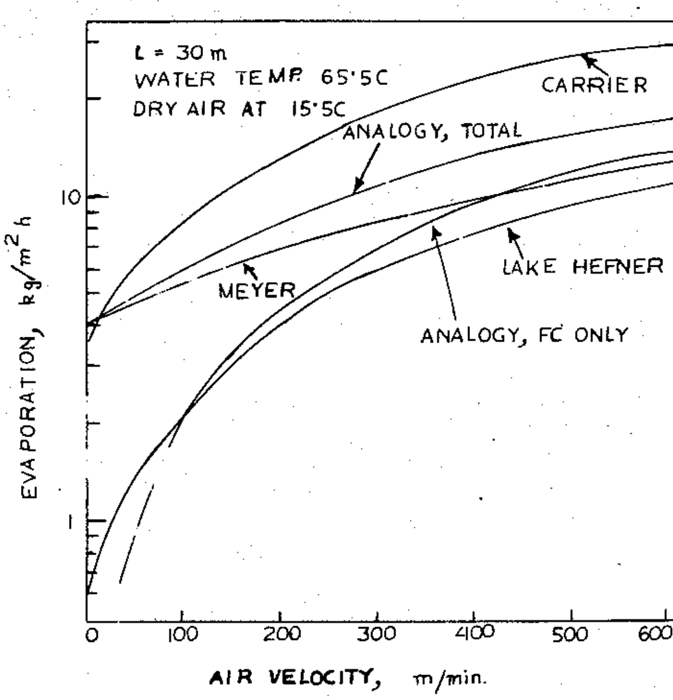


*Fig. 5 Effect of air velocity on the rate of evaporation calculated by various methods; water temperature is 65.5°C*

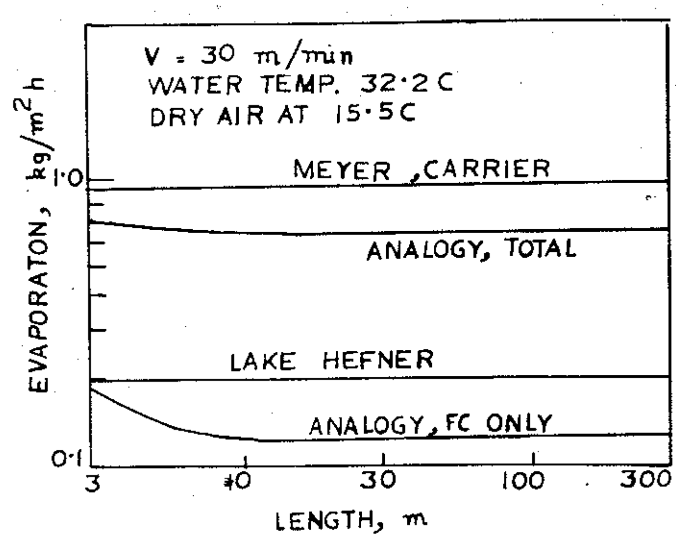


*Fig. 6 Effect of water surface length on the rate of evaporation calculated by various methods*

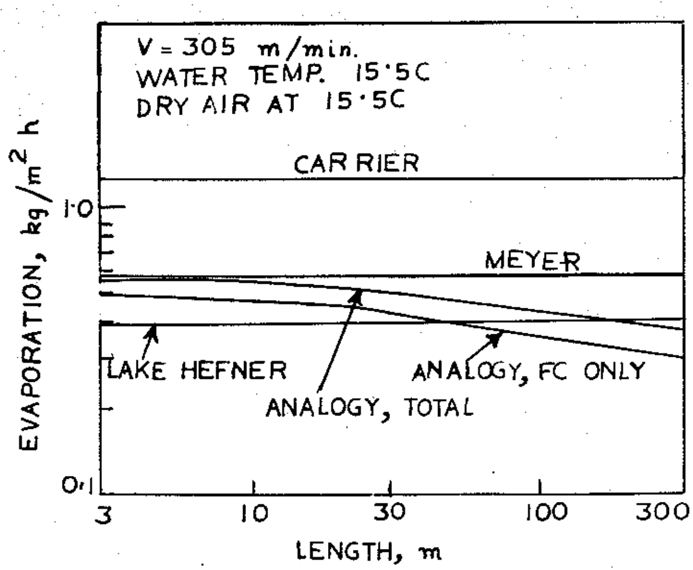


*Fig. 7 Effect of water surface length on the rate of evaporation calculated by various methods*

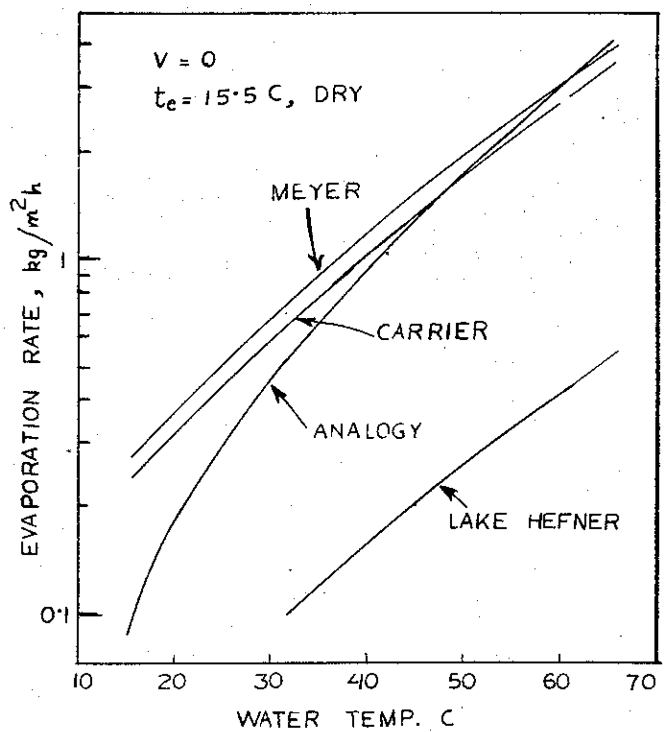


*Fig. 1 Evaporation rate at zero air velocity calculated by various methods; free stream air relative humidity is zero*

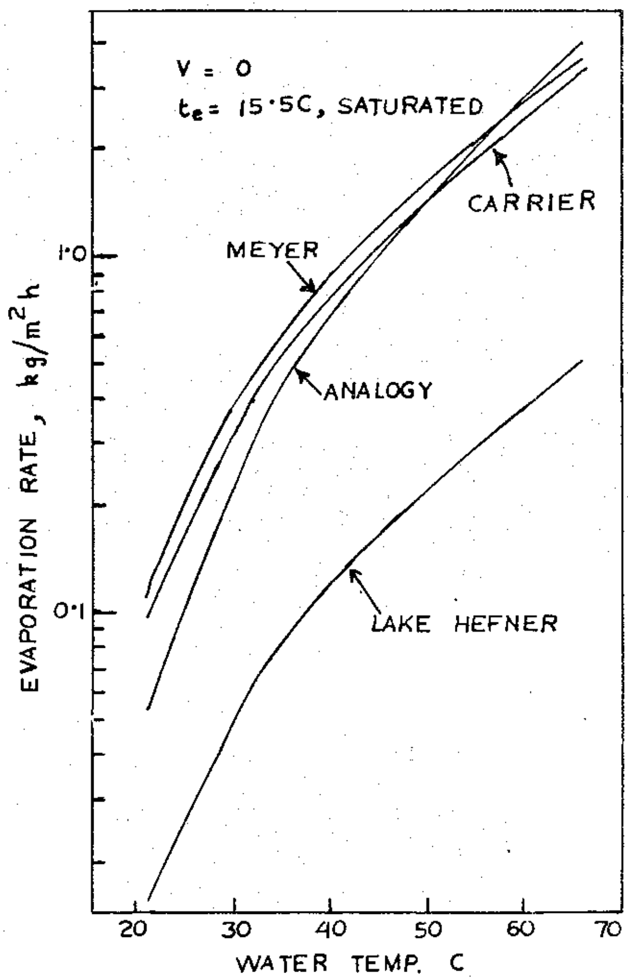


*Fig. 2 Evaporation rate at zero air velocity calculated by various methods; free stream air relative humidity is 100%*

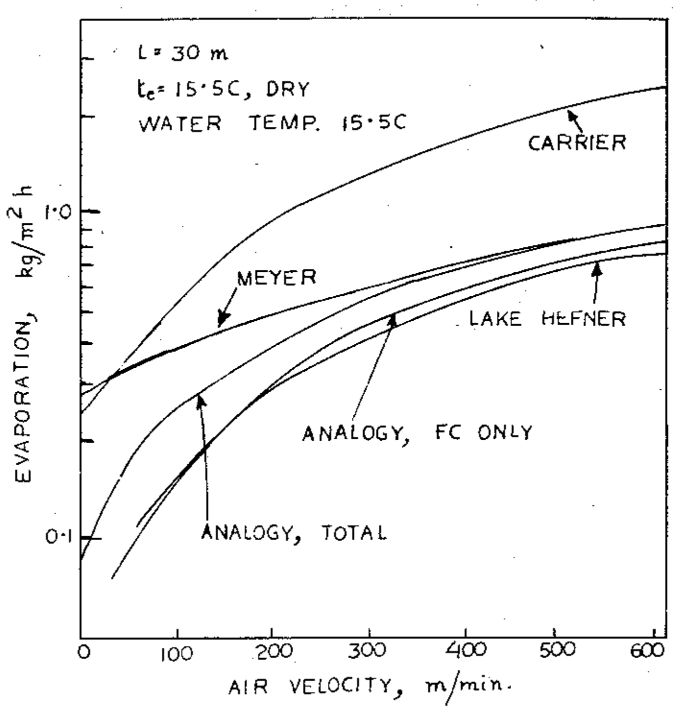


*Fig. 3 Effect of air velocity on the rate of evaporation calculated by various methods; water temperature is 15.5°C*

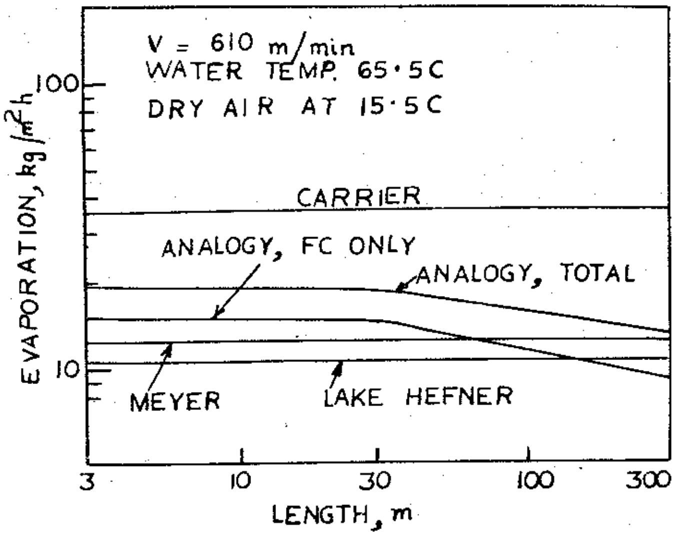


*Fig. 8 Effect of water surface length on the rate of evaporation calculated by various methods*

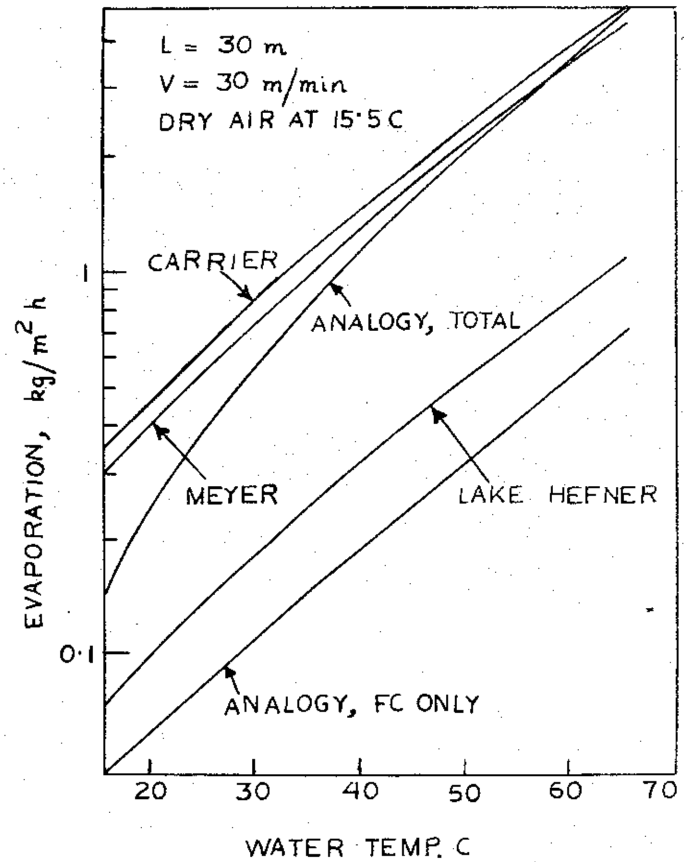


*Fig. 9 Effect of water temperature on the rate of evaporation calculated by various methods; air velocity is 30 m/min*

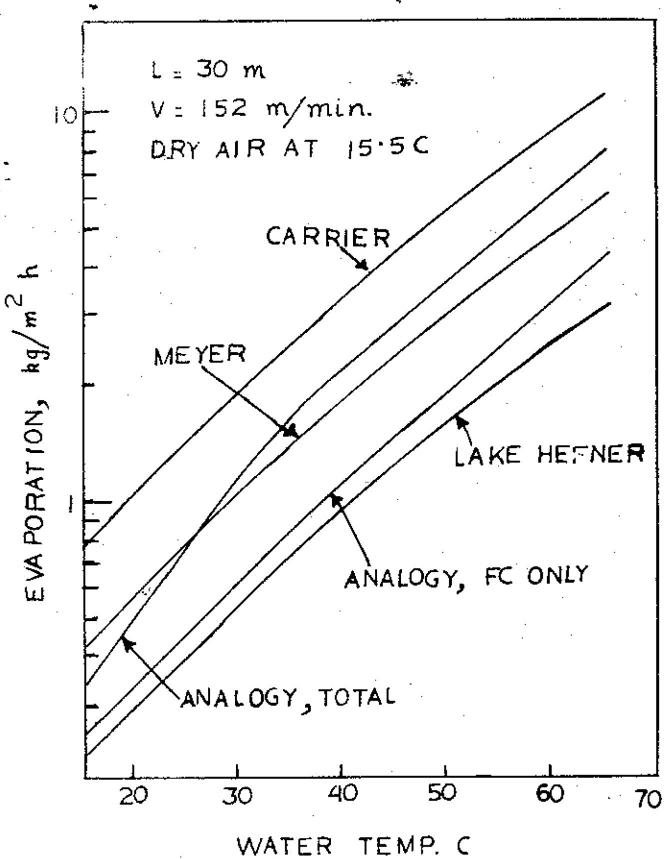


*Fig. 10 Effect of water temperature on the rate of evaporation calculated by various methods; air velocity is 152 m/min*

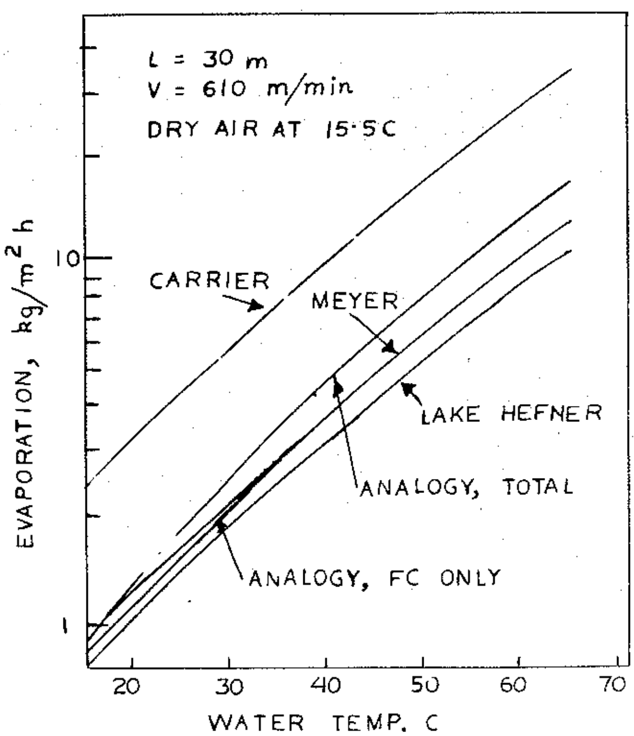


*Fig. 11 Effect of water temperature on the rate of evaporation calculated by various methods; air velocity is 610 m/min*